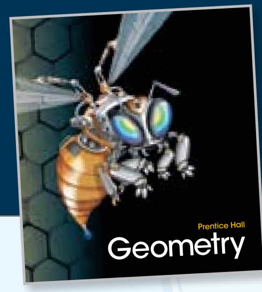
**Program:** Geometry SE, High School

**Scope:** Art development, photo research, page layout, feature page design, and design review for new on-level and foundations student editions.




CHAPTER 8

# Right Triangles and Trigonometry

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Does the height of the ramp affect the height of the jump? Sure it does. The ramp and its supports form a right triangle with the ground. The longer the support, the higher the ramp.

In this chapter, you'll find out how the sides of a right triangle are related.

## Vocabulary

English/Spanish Vocabulary Audio Online:

| English | Spanish |
|---|---|
| angle of depression, *p. 516* | ángulo de depresión |
| angle of elevation, *p. 516* | ángulo de elevación |
| cosine, *p. 507* | coseno |
| initial point, *p. 524* | punto inicial |
| magnitude, *p. 524* | magnitud |
| Pythagorean triple, *p. 492* | tripleta de Pitágoras |
| resultant, *p. 526* | vector resultante |
| sine, *p. 507* | seno |
| tangent, *p. 507* | tangente |
| terminal point, *p. 524* | punto terminal |
| vector, *p. 524* | vector |


## BIG ideas

1 **Measurement**
**Essential Question** How do you find a side length or angle measure in a right triangle?

2 **Proportionality**
**Essential Question** How do trigonometric ratios relate to similar right triangles?

3 **Coordinate Geometry**
**Essential Question** What is a vector?

## Chapter Preview

**Program:** Geometry SE, High School

**Scope:** Art development, photo research, page layout, feature page design, and design review for new on-level and foundations student editions.

Proof **Problem 1** Using the HL Theorem

**On the basketball backboard brackets shown below, $\angle ADC$ and $\angle BDC$ are right angles and $\overline{AC} \cong \overline{BC}$. Are $\triangle ADC$ and $\triangle BDC$ congruent? Explain.**

**Plan**

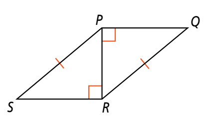
**How can you visualize the two right triangles?** Imagine cutting $\triangle ABC$ along $\overline{DC}$. On either side of the cut, you get triangles with the same leg $\overline{DC}$.


- You are given that $\angle ADC$ and $\angle BDC$ are right angles. So, $\triangle ADC$ and $\triangle BDC$ are right triangles.
- The hypotenuses of the two right triangles are $\overline{AC}$ and $\overline{BC}$. You are given that $\overline{AC} \cong \overline{BC}$.
- $\overline{DC}$ is a common leg of both $\triangle ADC$ and $\triangle BDC$. $\overline{DC} \cong \overline{DC}$ by the Reflexive Property of Congruence.

Yes, $\triangle ADC \cong \triangle BDC$ by the HL Theorem.

**Got It?** **1. a. Given:** $\angle PRS$ and $\angle RPQ$ are right angles, $\overline{SP} \cong \overline{QR}$

**Prove:** $\triangle PRS \cong \triangle RPQ$



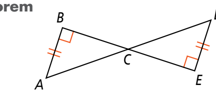

**b. Reasoning** Your friend says, "Suppose you have two right triangles with congruent hypotenuses and one pair of congruent legs. It does not matter which leg in the first triangle is congruent to which leg in the second triangle. The triangles will be congruent." Is your friend correct? Explain.



Proof **Problem 2** Writing a Proof Using the HL Theorem

**Given:** $\overline{BE}$ bisects $\overline{AD}$ at $C$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EC}$, $\overline{AB} \cong \overline{DE}$

**Prove:** $\triangle ABC \cong \triangle DEC$



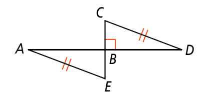

**Plan**

**How can you get started?** Identify the hypotenuse of each right triangle. Prove that the hypotenuses are congruent.

| Statement | Reason |
|---|---|
| $\overline{BE}$ bisects $\overline{AD}$. | Given |
| $\overline{AC} \cong \overline{DC}$ | Def. of bisector |
| $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EC}$ | Given |
| $\angle ABC$ and $\angle DEC$ are right $\angle$s. | Def. of $\perp$ lines |
| $\triangle ABC$ and $\triangle DEC$ are right $\triangle$s. | Def. of right triangle |
| $\overline{AB} \cong \overline{DE}$ | Given |
| $\triangle ABC \cong \triangle DEC$ | HL Theorem |

**Got It?** **2. Given:** $\overline{CD} \cong \overline{EA}$, $\overline{AD}$ is the perpendicular bisector of $\overline{CE}$

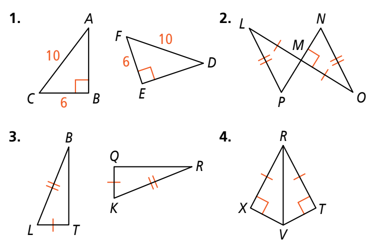
**Prove:** $\triangle CBD \cong \triangle EBA$




## Lesson Check

### Do you know HOW?

**Are the two triangles congruent? If so, write the congruence statement.**

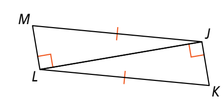



### Do you UNDERSTAND?

5. **Vocabulary** A right triangle has side lengths of 5 cm, 12 cm, and 13 cm. What is the length of the hypotenuse? How do you know?

6. **Compare and Contrast** How do the HL Theorem and the SAS Postulate compare? How are they different? Explain.

7. **Error Analysis** Your classmate says that there is not enough information to determine whether the two triangles below are congruent. Is your classmate correct? Explain.



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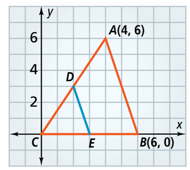
**Here's Why It Works** You can verify that the Triangle Midsegment Theorem works for a particular triangle. Use the following steps to show that $\overline{DE} \parallel \overline{AB}$ and that $DE = \frac{1}{2}AB$ for a triangle with vertices at $A(4, 6)$, $B(6, 0)$, and $C(0, 0)$, where $D$ and $E$ are the midpoints of $\overline{CA}$ and $\overline{CB}$.




**Step 1** Use the Midpoint Formula, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, to find the coordinates of $D$ and $E$.

The midpoint of $\overline{CA}$ is $D\left(\frac{0+4}{2}, \frac{0+6}{2}\right) = D(2, 3)$.

The midpoint of $\overline{CB}$ is $E\left(\frac{0+6}{2}, \frac{0+0}{2}\right) = E(3, 0)$.

**Step 2** To show that the midsegment $\overline{DE}$ is parallel to the side $\overline{AB}$, find the slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$, of each segment.

slope of $\overline{DE} = \frac{0-3}{3-2} = \frac{-3}{1} = -3$   slope of $AB = \frac{0-6}{6-4} = \frac{-6}{2} = -3$

**Step 3** To show $DE = \frac{1}{2}AB$, use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find $DE$ and $AB$.

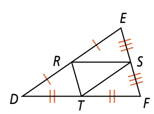
$DE = \sqrt{(3-2)^2 + (0-3)^2} = \sqrt{1+9} = \sqrt{10}$   $AB = \sqrt{(6-4)^2 + (0-6)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

Since $\sqrt{10} = \frac{1}{2}(2\sqrt{10})$, you know that $DE = \frac{1}{2}AB$.

**Think**
**How do you identify a midsegment?**
Look for indications that the endpoints of a segment are the midpoints of a side of the triangle.

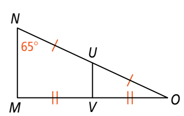
**Problem 1** Identifying Parallel Segments

**What are the three pairs of parallel segments in $\triangle DEF$?**




$\overline{RS}$, $\overline{ST}$, and $\overline{TR}$ are the midsegments of $\triangle DEF$. By the Triangle Midsegment Theorem, $\overline{RS} \parallel \overline{DF}$, $\overline{ST} \parallel \overline{ED}$, and $\overline{TR} \parallel \overline{FE}$.

**Got It?** 1. a. In $\triangle XYZ$, $A$ is the midpoint of $\overline{XY}$, $B$ is the midpoint of $\overline{YZ}$, and $C$ is the midpoint of $\overline{ZX}$. What are the three pairs of parallel segments?
b. **Reasoning** What is $m\angle VUO$ in the figure at the right? Explain your reasoning.

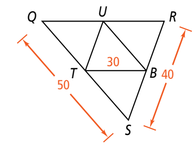





**Plan**
**Which relationship stated in the Triangle Midsegment Theorem should you use?**
You are asked to find lengths, so use the relationship that refers to the lengths of a midsegment and the third side.

**Problem 2** Finding Lengths

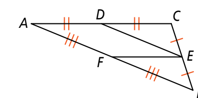
**In $\triangle QRS$, $T$, $U$, and $B$ are midpoints. What are the lengths of $\overline{TU}$, $\overline{UB}$, and $\overline{QR}$?**




Use the relationship
length of a midsegment $= \frac{1}{2}$(length of the third side)
to write an equation about the length of each midsegment.

$TU = \frac{1}{2}SR = \frac{1}{2}(40) = 20$   $UB = \frac{1}{2}QS = \frac{1}{2}(50) = 25$   $TB = \frac{1}{2}QR$; $30 = \frac{1}{2}QR$; $60 = QR$

**Got It?** 2. In the figure at the right, $AD = 6$ and $DE = 7.5$. What are the lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$?




You can use the Triangle Midsegment Theorem to find lengths of segments that might be difficult to measure directly.

**Think**
**Why does the geologist find the length of $\overline{CD}$?**
$\overline{CD}$ is a midsegment of $\triangle AEB$, so the geologist can use its length to find $AB$, the distance across the sinkhole.

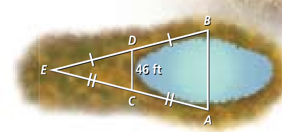
**Problem 3** Using a Midsegment of a Triangle

**Environmental Science** **A geologist wants to determine the distance, $AB$, across a sinkhole. Choosing a point $E$ outside the sinkhole, she finds the distances $AE$ and $BE$. She locates the midpoints $C$ and $D$ of $\overline{AE}$ and $\overline{BE}$ and then measures $\overline{CD}$. What is the distance across the sinkhole?**

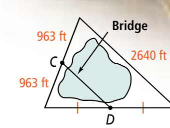
$CD$ is a midsegment of $\triangle AEB$.

$CD = \frac{1}{2}AB$   △ Midsegment Thm.
$46 = \frac{1}{2}AB$   Substitute 46 for $CD$.
$92 = AB$   Multiply each side by 2.

The distance across the sinkhole is 92 ft.




**Got It?** 3. $\overline{CD}$ is a bridge being built over a lake, as shown in the figure at the right. What is the length of the bridge?



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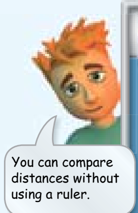
# 5-7 Inequalities in Two Triangles

**Sunshine State Standard**
**MA.912.G.4.7** Apply the inequality theorems: triangle inequality and the Hinge Theorem.

**Objective** To apply inequalities in two triangles




**Getting Ready!**

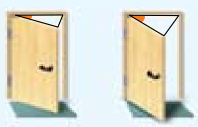
Think of a clock or watch that has an hour hand and a minute hand. As minutes pass, the distance between the tip of the hour hand and the tip of the minute hand changes. This distance is x in the figure at the right. What is the order of the times below from least to greatest length of x? How do you know?

1:00, 3:00, 5:00, 8:30, 1:30, 12:20

In the Solve It, the hands of the clock and the segment labeled *x* form a triangle. As the time changes, the shape of the triangle changes, but the lengths of two of its sides do not change.

**Essential Understanding** In triangles that have two pairs of congruent sides, there is a relationship between the included angles and the third pair of sides.

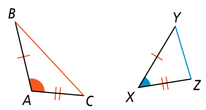
When you close a door, the angle between the door and the frame (at the hinge) gets smaller. The relationship between the measure of the hinge angle and the length of the opposite side is the basis for the SAS Inequality Theorem, also known as the Hinge Theorem.



**take note**

### Theorem 5-13 The Hinge Theorem (SAS Inequality Theorem)

| Theorem | If . . . | Then . . . |
|---|---|---|
| If two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle. | $m\angle A > m\angle X$ | $BC > YZ$ |



*You will prove Theorem 5-13 in Exercise 25.*

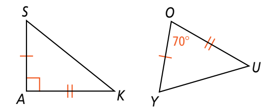


**Problem 1** Using the Hinge Theorem

**Plan**
**How do you apply the Hinge Theorem?** After you identify the angles included between the pairs of congruent sides, locate the sides opposite those angles.

**Multiple Choice** Which of the following statements must be true?

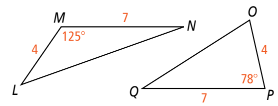
Ⓐ $AS < YU$ Ⓒ $SK < YU$
Ⓑ $SK > YU$ Ⓓ $AK = YU$



$\overline{SA} \cong \overline{YO}$ and $\overline{AK} \cong \overline{OU}$, so the triangles have two pairs of congruent sides. The included angles, $\angle A$ and $\angle O$, are not congruent. Since $m\angle A > m\angle O$, $SK > YU$ by the Hinge Theorem. The correct answer is B.

**Got It?** 1. a. What inequality relates $LN$ and $OQ$ in the figure at the right?

b. **Reasoning** In $\triangle ABC$, $AB = 3$, $BC = 4$, and $CA = 6$. In $\triangle PQR$, $PQ = 3$, $QR = 5$, and $RP = 6$. How can you use indirect reasoning to explain why $m\angle P > m\angle A$?

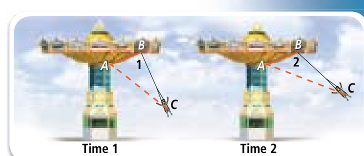


**Problem 2** Applying the Hinge Theorem

**Think**
**For $\triangle ABC$, which side lengths are the same at Time 1 and Time 2?** The length of the chain and the distance between the top of the chain and the top of the column do not change. So, $AB$ and $BC$ are the same at Time 1 and Time 2.

**Swing Ride** As the speed of the swing ride increases, the angle between the chain and the top of the column increases. The diagram below shows the position of a swing at two different times. Is the rider farther from the top of the column at Time 1 or Time 2? Explain how the Hinge Theorem justifies your answer.

The rider is farther from the top of the column at Time 2. The lengths of $\overline{AB}$ and $\overline{BC}$ stay the same throughout the ride. Since the angle formed at Time 2 ($\angle 2$) is greater than the angle formed at Time 1 ($\angle 1$), you can use the Hinge Theorem to conclude that $\overline{AC}$ at Time 2 is longer than $\overline{AC}$ at Time 1.



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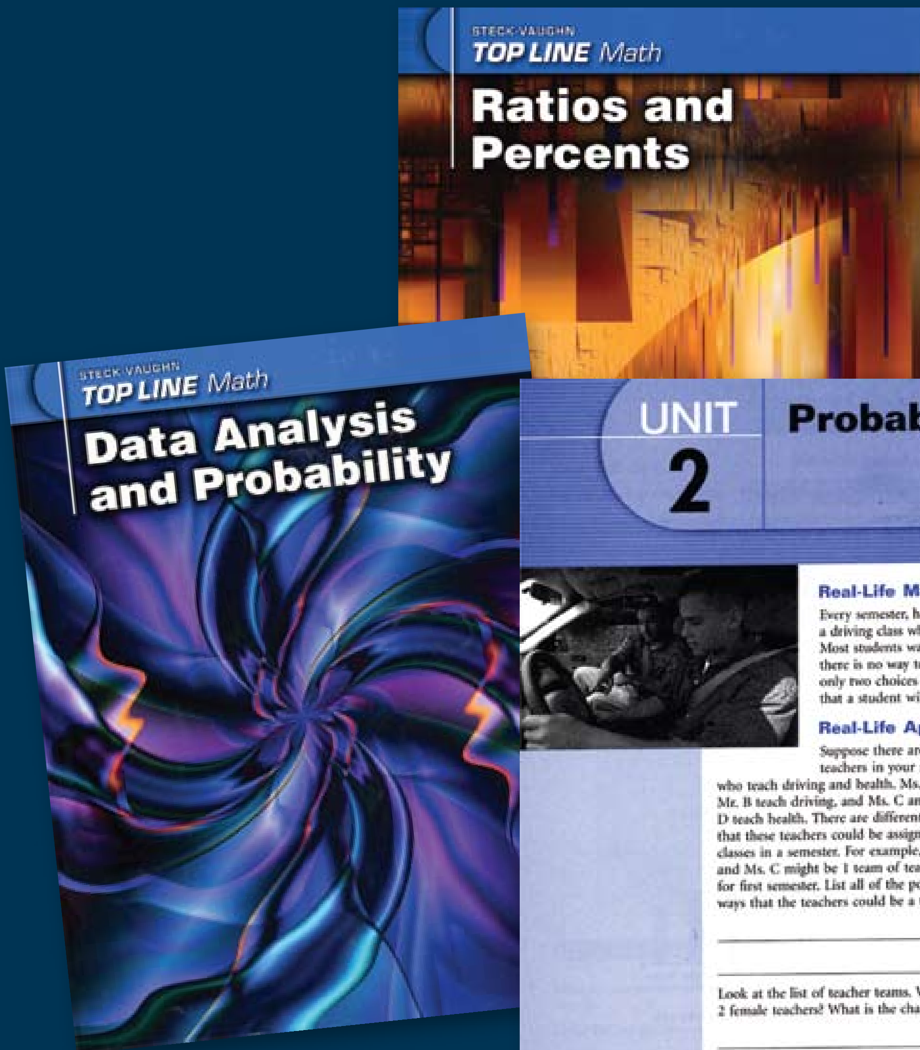


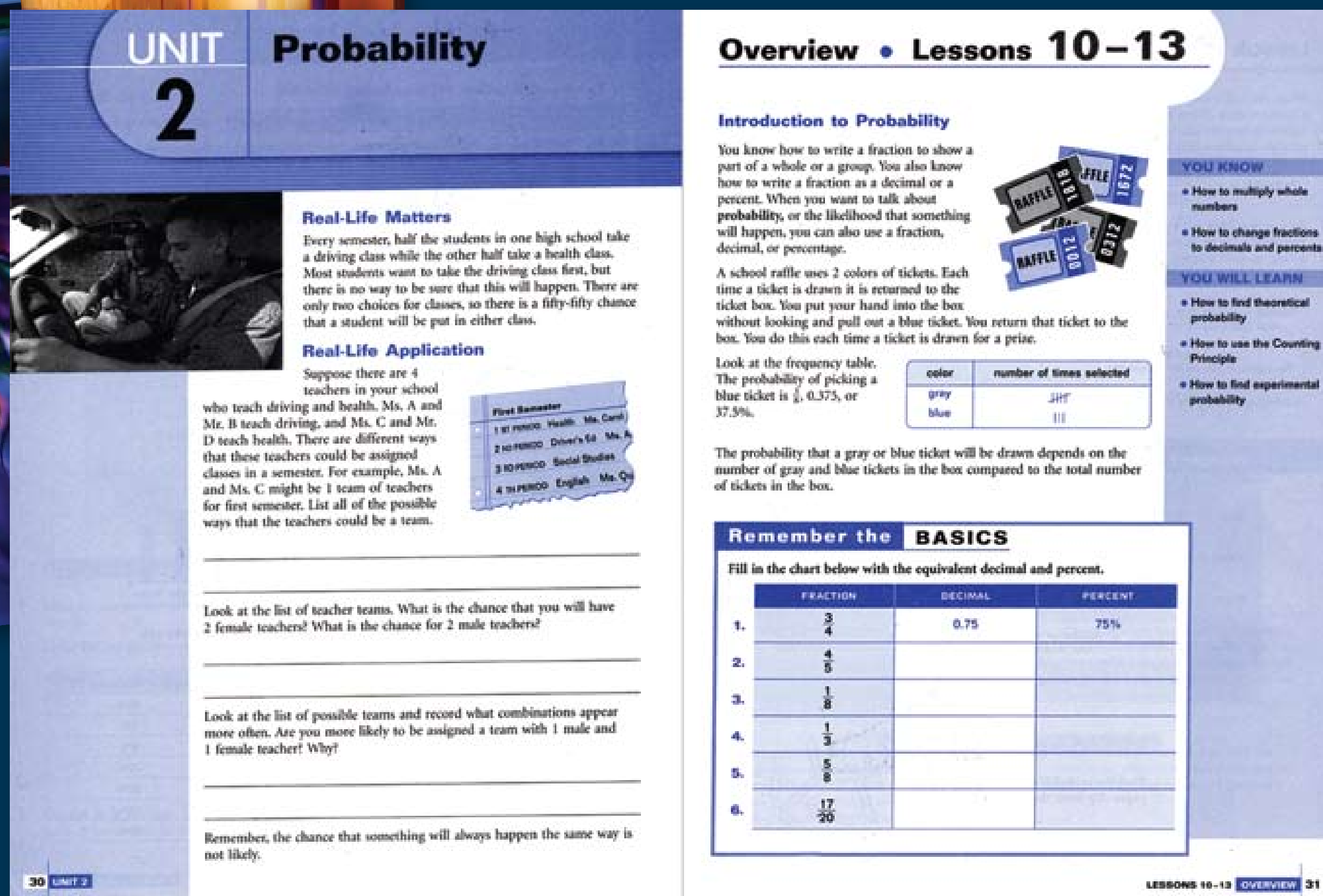

UNIT 2

# Probability

## Real-Life Matters

Every semester, half the students in one high school take a driving class while the other half take a health class. Most students want to take the driving class first, but there is no way to be sure that this will happen. There are only two choices for classes, so there is a fifty-fifty chance that a student will be put in either class.

## Real-Life Application

Suppose there are 4 teachers in your school who teach driving and health. Ms. A and Mr. B teach driving, and Ms. C and Mr. D teach health. There are different ways that these teachers could be assigned classes in a semester. For example, Ms. A and Ms. C might be 1 team of teachers for first semester. List all of the possible ways that the teachers could be a team.

Look at the list of teacher teams. What is the chance that you will have 2 female teachers? What is the chance for 2 male teachers?

Look at the list of possible teams and record what combinations appear more often. Are you more likely to be assigned a team with 1 male and 1 female teacher? Why?

Remember, the chance that something will always happen the same way is not likely.



# Overview • Lessons 10–13

## Introduction to Probability

You know how to write a fraction to show a part of a whole or a group. You also know how to write a fraction as a decimal or a percent. When you want to talk about **probability**, or the likelihood that something will happen, you can also use a fraction, decimal, or percentage.

A school raffle uses 2 colors of tickets. Each time a ticket is drawn it is returned to the ticket box. You put your hand into the box without looking and pull out a blue ticket. You return that ticket to the box. You do this each time a ticket is drawn for a prize.

Look at the frequency table. The probability of picking a blue ticket is $\frac{3}{8}$, 0.375, or 37.5%.

| color | number of times selected |
|---|---|
| gray | ~~\|\|\|\|~~ |
| blue | \|\|\| |

The probability that a gray or blue ticket will be drawn depends on the number of gray and blue tickets in the box compared to the total number of tickets in the box.

**YOU KNOW**

- How to multiply whole numbers
- How to change fractions to decimals and percents

**YOU WILL LEARN**

- How to find theoretical probability
- How to use the Counting Principle
- How to find experimental probability

## Remember the BASICS

Fill in the chart below with the equivalent decimal and percent.

| | FRACTION | DECIMAL | PERCENT |
|---|---|---|---|
| 1. | $\frac{3}{4}$ | 0.75 | 75% |
| 2. | $\frac{4}{5}$ | | |
| 3. | $\frac{1}{8}$ | | |
| 4. | $\frac{1}{3}$ | | |
| 5. | $\frac{5}{8}$ | | |
| 6. | $\frac{17}{20}$ | | |